Mean squares

- Take two positive values > 1
- Find the mean of the two values
- Square it

Then

- Take the same two values
- Square them
- Find the mean of the squares

Which value is greater?
Is this always true?
Can you prove it?

- Try out several examples
- Is one expression always bigger than the other?
- Next try using x and y instead.
- If you subtract one expression from the other, can you work out if it's positive or negative?

Mean squares Solution

- Take two positive values > 1 → $x \; and \; y$
- Find the mean of the two values → $\frac{x+y}{2}$
- Square it → $\left(\frac{x+y}{2}\right)^2 = \frac{x^2+y^2+2xy}{4}$

Then

- Take the same two values → $x \; and \; y$
- Square them → $x^2 and \; y^2$
- Find the mean of the squares → $\frac{x^2+y^2}{2}$

Which value is greater?
Is this always true?
Can you prove it?

Mean squares Solution

- Take two positive values > 1
- Find the mean of the two values
- Square it

Then

- Take the same two values
- Square them
- Find the mean of the squares

→ $\frac{x^2 + y^2 + 2xy}{4}$

→ $\frac{x^2 + y^2}{2}$

- If you subtract one expression from the other, can you work out if it's positive or negative?

$$\frac{x^2+y^2}{2} - \frac{x^2+y^2+2xy}{4}$$

$$= \frac{2x^2+2y^2}{4} - \frac{x^2+y^2+2xy}{4}$$

$$= \frac{x^2 + y^2 - 2xy}{4}$$

$$= \frac{(x-y)^2}{4}$$

Since we know any square number is positive, we can say

$$\frac{(x-y)^2}{4} > 0$$

Therefore squaring the mean is greater than the mean of the squares