Difference of numeric squares

Problem 1:

Mrs Gryce was asked to calculate 18×12 by Mr Lo who had forgotten his calculator and was doing some marking.

Mrs Gryce quickly responded

'Well, that's just $15^2 - 9$ which is 216'

Mr Lo was amazed.

- How did she know so quickly what the answer was?

Problem 1 Solution

We want to show that $18 \times 12 = 15^2 - 9$

Using our knowledge of the Difference of Two Squares

$$\begin{aligned} 18 \times 12 &= (15 + 3) \times (15 - 3) \\ &= 15^2 - 3^2 \\ &= 15^2 - 9 \end{aligned}$$

Why 15? 15 is midway between 12 and 18

Why 3? 15 ± 3 is 12 and 18

Also 15^2 and 3^2 are straightforward to work out.

- How could you use this to calculate

a) 29x21 b) 6x14 c) 19x21

Would you use this to calculate 73 x 78?

Difference of numeric squares

Problem 2:

- Use the fact that 3 x 4 = 12
- Can you quickly work out a value for $(3.5)^2$?

Can you see a connection between the previous question and this one?

Problem 2 Solution

We can also use the Difference of Two Squares in reverse to help us square numbers

$$3 \times 4 = 12$$

$$(3.5 - 0.5)(3.5 + 0.5) = 12$$

$$3.5^2 - 0.5^2 = 12$$

$$3.5^2 = 12 + 0.5^2$$

$$3.5^2 = 12.25$$

We are effectively starting at 3.5 and stepping an equal distance in both directions along the number line.

Important! Understanding the Difference of Two Squares from the point of view of an 'averaging' of values.

We can use this to calculate any $n.5^2$, for example when n=6,

$$6.5^2 = 6 \times 7 + 0.25$$

Can you see how $2.25^2 = 2 \times 2.5 + 0.0625$?