Sketching Solutions

Linear Sketching

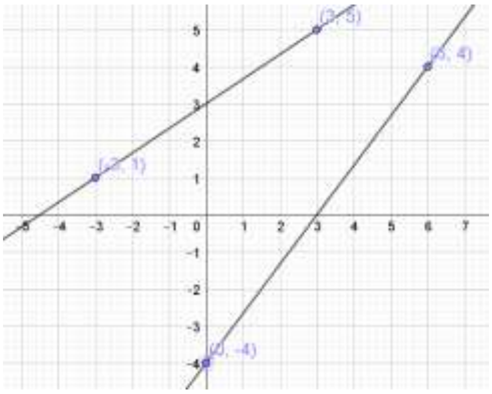
Linear Graphs 1

1. Gradient = 3, intercept = -5
2. Gradient = $\frac{10-6}{3-1} = 2$
3. Midpoint = $(\frac{3+-1}{2}, \frac{-8+4}{2}) = (1, \ -2)$
4. Distance = $\sqrt{(4-1)^2+(18-10)^2} = \sqrt{73}$
5. Equation is $y = 3x - 7$
6. $y = 2 \times 1 \ - 3 = -1$
   a. Yes the line passes through (1,-1)
7. Equation is y=5x+9
8. $y = 3x - 1$

Linear Graphs 2

1. Gradient = 2, intercept = -7
2. Gradient = $\frac{4-0}{1--1} = 2$
3. Midpoint = $(\frac{-2+6}{2}, \frac{10+4}{2}) = (2,7)$
4. Distance = $\sqrt{(4--1)^2+(11-15)^2} = \sqrt{41}$
5. Equation is $y = 2x + 2$
6. No, the line doesn't pass through (3,1) as when $x = 3, y = -1$
7. Equation is $y = -\frac{3}{2}x + 13$
8. $y = \frac{1}{2}x + 2$

Do they cross?



From a sketch we can see that the lines are not parallel.

They will meet at some point

The equations of the lines are $y = \frac{4}{3}x - 4$ and $y = \frac{2}{3}x + 3$

Solving simultaneously we find that the lines intersect at (10.5, 10)

Sketching Solutions

Picture this

$x + 2y = 1$ should have a negative gradient, which it doesn't in the sketch
Also, the y intercept is $\left(0, \frac{1}{2}\right)$, the x intercept is $\left(\frac{1}{1}, 0\right) = (1,0)$
So they have sketched $-x + 2y = 1$

$2x + 5y = 10$ should have a negative gradient, which it does.
The y intercept is $\left(0, \frac{10}{5}\right) = (0,2)$ and the x intercept is $\left(\frac{10}{2}, 0\right) = (5,0)$
So this line is correct

The plot thickens….

Name	Equation	x intercept	y intercept	Positive/negative gradient
A	$y - 2x - 1 = 0$	$(-\frac{1}{2}, 0)$	$(0, 1)$	Positive
B	$y = 3$	No intercept	$(0, 3)$	Horizontal line
C	$3x + 4y = 2$	$(\frac{2}{3}, 0)$	$(0, \frac{1}{2})$	Negative
D	$2x - y + 6 = 0$	$(-3, 0)$	$(0, 6)$	Positive
E	$2y + x = 4$	$(4, 0)$	$(0, 2)$	Negative
F	$2x + y - 3 = 0$	$(\frac{3}{2}, 0)$	$(0, 3)$	Negative

A pair of lines that are parallel — A and D as they do not intersect

A pair of lines that are perpendicular — A and E or D and E

A pair of lines that intersect at $(-2, 2)$ — C and D

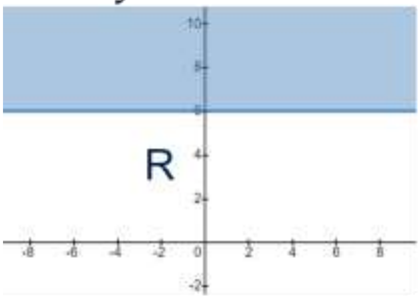
Two geometry problems

The equation of line DE	$y = 3x - 15$	The coordinate of C	$(7,7)$
The possible coordinates of F	$(3,4)$ or $(9,2)$	The equation of line AB	$y = 4$
The equation of line EF	$y = \frac{1}{2}x - \frac{5}{2}$ or	The equation of line BD	$y = -\frac{3}{2}x + 10$
	$y = -2x + 10$	The area of the parallelogram	15 units^2

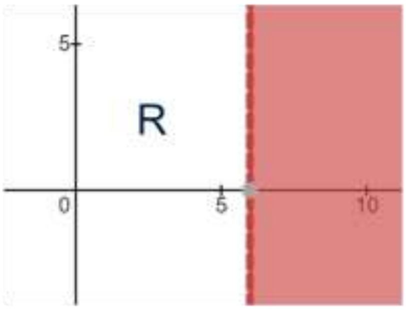
Sketching Solutions

Sketching Linear Inequalities

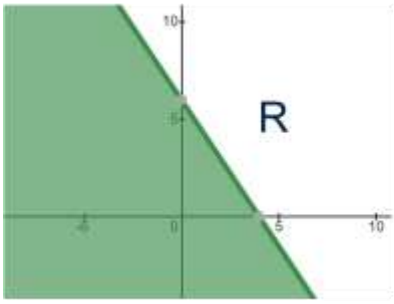
1. $y \leq 6$



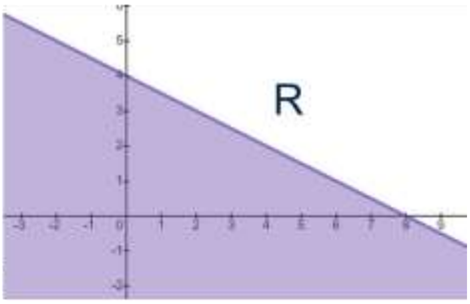
2. $x < 6$



3. $x + 2y \geq 8$



4. $3x + 2y \geq 12$



Geometry from equations

Which are the two pairs of parallel sides? $y - 2 = x$ with $y = x - 1$ and $y + x = 6$ with $y + x - 3 = 0$

What are the coordinates of all 4 vertices? (0.5, 2.5) (2, 4) (3.5, 2.5) (2, 1)

How can you convince yourself this is a square? As well as all the lines that meet being perpendicular, you also need to show they all have the same length. You can do this by using Pythagoras' theorem, or calculating the column vector.

Linear Programming

To maximise the value of $x + y$ within the feasible region, we substitute in the coordinates of each vertex.

$$(0,6) \quad x + y = 0 + 6 = 6$$
$$(2,3) \quad x + y = 2 + 3 = 5$$
$$(6,6) \quad x + y = 6 + 6 = 12$$
$$(6,1) \quad x + y = 6 + 1 = 7$$

So the **maximum value of $x + y$ is 12** at the point (6,6)

Quadratic Sketching

Sketching Solutions

Quadratic Graphs 1

1. A $(2,0)$
B $(-3,0)$
C $(0,-6)$

2. A $(3,0)$
B $(-2,0)$
C $(0,-6)$

3. A $(5,0)$
B $(-1,0)$
C $(0,-5)$

4. A $(4,0)$
B $(-5,0)$
C $(0,-20)$

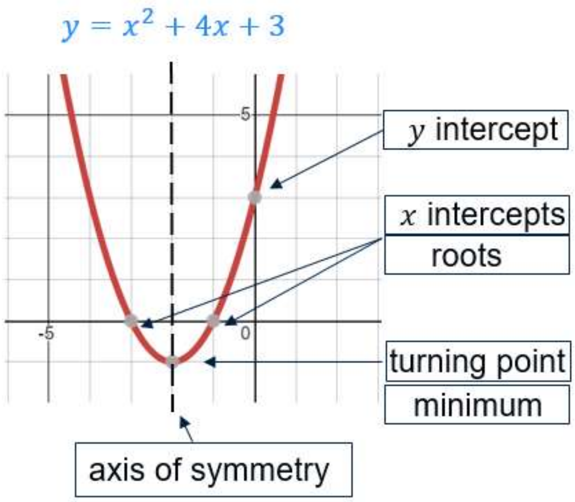
5. C $(0,5)$
D $(1,4)$

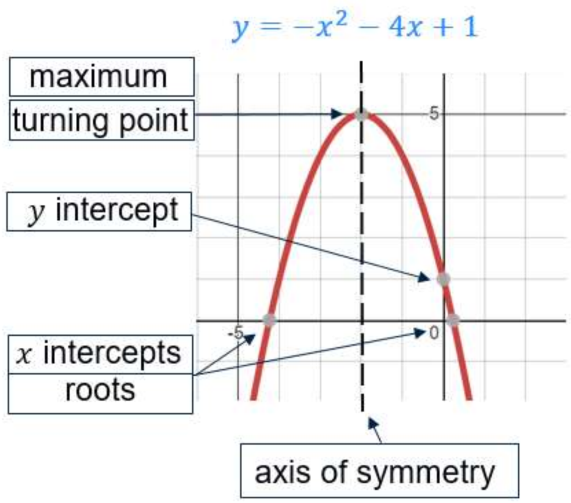
6. C $(0,16)$
D $(-3,7)$

7. A $(1,0)$
B $(-4,0)$
C $(0,-4)$

8. A $(1,0)$
B $(-3,0)$
C $(0,-6)$
D $(-1,-8)$

What is a sketch?





Identification Parade

Graph C is $\boldsymbol{y = x^2 - 5x + 4}$**?**

Move it!

Graph A – Graph B. Translate by the vector $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

Graph A – Graph C. Translate by the vector $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

Graph A – Graph D. Translate by the vector $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

Graph A – Graph E. Translate by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Graph A – Graph F. Translate by the vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

Sketching Solutions

Complete the square to get sorted!

$y = x^2 + 8x + 20$ $y = (x+4)^2 + 4$ $(-4,4)$	$y = x^2 - 2x + 5$ $y = (x-1)^2 + 4$ $(1,4)$	$y = x^2 - 10x + 29$ $y = (x-5)^2 + 4$ $(5,4)$
$y = x^2 + 6x + 10$ $y = (x+3)^2 + 1$ $(-3,1)$	$y = x^2 - 4x + 7$ $y = (x-2)^2 + 3$ $(2,3)$	$y = x^2 + 2x + 5$ $y = (x+1)^2 + 4$ $(-1,4)$
$y = x^2 - 6x + 16$ $y = (x-3)^2 + 7$ $(3,7)$	$y = x^2 - 6x + 11$ $y = (x-3)^2 + 2$ $(3,2)$	$y = x^2 - 6x + 25$ $y = (x-3)^2 + 16$ $(3,16)$

Quadratic Graphs 2

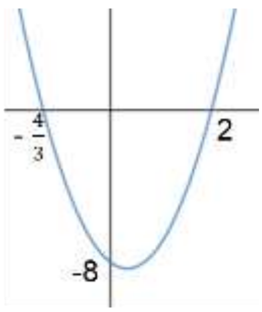
1. x intercepts are $\left(-\frac{3}{2},\ 0\right)$ and $(-4,0)$

2. x intercepts are $(-3,\ 0)$and $(2,0)$
   y intercepts is $(0,-6)$

3. $y = x^2 - x\ - 6$

4. x intercepts are $\left(-\frac{2}{3},0\right)$and $(\frac{1}{2},0)$

5. The y intercept. Coordinate $(0,c)$

6. 

7. C is (0,16) D is $(-3{,}7)$

8.

Has a minimum point at (2, 4)	Will not cross the x axis twice

Sketching Solutions

How High?

The ball hits the ground when t=2 (after 2 seconds)

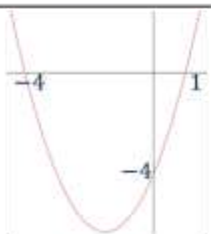
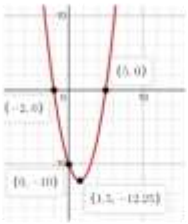
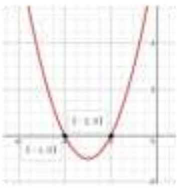
The ball is more than 5m above the ground for 1.414 seconds (to 2dp)

The maximum height reached by the ball is 10 metres

Quadratic Inequalities

1. $-3 < x < 2$ 2. $x < -4 \text{ or } x > 2$ 3. $x \leq -4 \text{ or } x \geq -3$ 4. $-8 \leq x \leq 4$

Fill the table

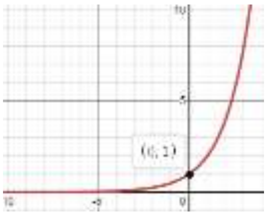
Sketch	Equation	x intercept	y intercept	Minimum point
	$y=(x+4)(x-1)$ $y=x^2+3x-4$	$(-4,0)(1,0)$	$(0,-4)$	$\left(x+\frac{3}{2}\right)^2-\frac{9}{4}-4$ $\left(x+\frac{3}{2}\right)^2-\frac{25}{4}$ Min point $(-\frac{3}{2},-\frac{25}{4})$
	$y=(x-5)(x+2)$ $y=x^2-3x-10$	$(5,0)\ (-2,0)$	$(0,-10)$	$\left(x-\frac{3}{2}\right)^2-\frac{9}{4}-10$ $\left(x-\frac{3}{2}\right)^2-\frac{49}{4}$ Min point $(\frac{3}{2},-\frac{49}{4})$
	$y=(x+5)^2+6$ $y=x^2+10x+31$	Sits above x axis so no intercepts	$(0,31)$	(-5, 6)
	$y=x^2+6x+8$	$(-4,0)(-2,0)$	$(0,8)$	$(x+3)^2-9+8$ $(x+3)^2-1$ Min point $(-3,-1)$

Sketching Solutions

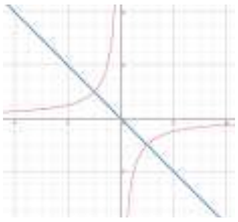
Sketching Other Graphs

Sketching Other Graphs 1

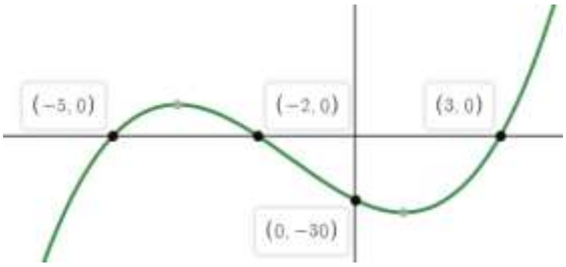
1. A reciprocal Graph
2. Max value = 1 Min value = -1
3. As x gets very large y gets very large. As x gets very small, y tends to zero but stays positive.



4. There will be two solutions

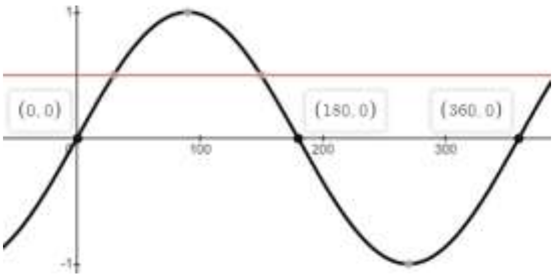


5. A cubic
6. y intercept at $(0, -30)$
7. x intercepts at $(-2,0)$ $(3,0)$ $(-5,0)$
8.

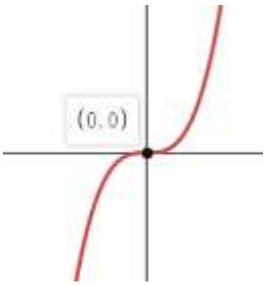


Sketching Other Graphs 2

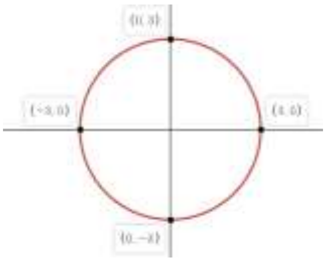
1. Circles are of the form $x^2 + y^2 = r^2$
2.



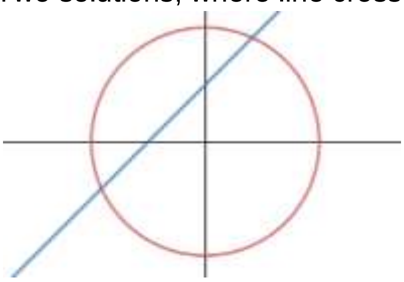
3. Two solutions $30°$ and $150°$, the points of intersection above
4.



5.



6. y intercept is $(0, -1)$
7. x intercept is $(-1,0)$ repeated and $(1,\ 0)$
8. Two solutions, where line crosses circle



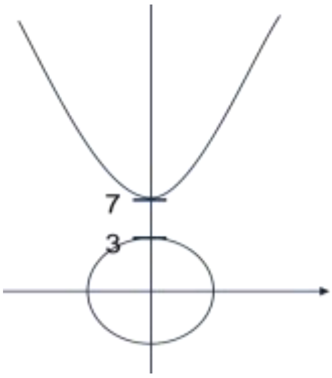
Sketching Solutions

Which is which?

A $y = 3^x$ **B** $y = x^2$ **C** $y^2 + x^2 = 16$ **D** $y = \frac{3}{x}$

E $y = x^3$ **F** $y = \sin(x)$

Shortest distance



From the sketch we can see that the graphs are separated at the two marked points and this will be the shortest distance, which is 4 units

How fast?

a) 350m/min

b) 800m/min

A square in a circle

The ratio of C_1 to C_2 is $\sqrt{\mathbf{2}} : \mathbf{1}$

A Triggy Problem!

Fortunately, this is an already factorised quadratic. So, $x = 270°$ or $x = 60°\ and\ 300°$

A cubic match up

$\boldsymbol{y = x(x-1)(x+1)}$