

# Percentages

## Problems for the

### 11-14 classroom

# Teacher Guidance

A collection of problems from  
the FMSP and the UKMT  
to support teachers in  
11-14 classrooms



## Problems within the classroom: Teacher Guidance

Welcome to our set of problems on percentages.

You can use these problems in a variety of ways: you could choose one as a starter, as a plenary or a review of previous knowledge; but we would suggest using all of them as the main focus of a lesson to help develop problem solving skills.

To add an element of friendly competition you may wish to put the class into teams and allow them to choose the problem on which they start... you could support students by giving them 'hint tokens' which they can exchange for hints when they get stuck – but only if the whole team has thought about it!

You could offer points for correct answers with extra points for elegant, efficient solutions and offer prizes such as merit points for the team which gets the most points.

Each problem has been created with the hints and solutions included in this guidance. We offer these problems in alphabetical order rather than an order of difficulty or a teaching order as these would depend on your knowledge of your students.

There are pages in the booklet which can be photocopied to provide the problems in an easy to display / distribute format with hints at the end which can be cut up to allow students to have one hint at a time.

All of these problems can be tackled without a calculator, but you may use them to support your students if you wish.

*Our aim is to provide problems which support all students in all schools.*



## Bus travel...

**Topics covered:** Using percentages of amounts

**Prerequisite knowledge:** Basic understanding of percentages, finding percentages of amounts

**Problem:**

36% of the students at a certain school are mainly left-handed and all the other students are mainly right-handed. 25% of the left-handed students and 50% of the right-handed students at this school travel to school by bus. What percentage of the whole school travel by bus?

**Hints to support students:**

Easier:

If there are 1000 students in the school how many left-handed students are there?

How many left-handed students travel to school by bus?

How many right-handed students travel to school by bus?

OR:

Harder:

Pick a number of students for there to be in the school. Now find the number of left-handed students in the school.

What do you think you might find out next?

**Solution:**

25% of 36% is 9% so 9% of the students are left-handed students who travel by bus

If 36% are left-handed and the rest are right-handed then 64% are right-handed

50% of 64% is 32% so 32% of students are right-handed students who travel by bus

9% + 32% is 41% so 41% of students travel by bus



## Giving orders...

**Topics covered:** Order of percentages

**Prerequisite knowledge:** Finding a percentage of an amount

### **Problem:**

The Grand Old Duke of York had 10 000 men. He lost 10% of them on the way to the top of the hill and he lost 15% of the rest as he marched them back down the hill. The Grand Young Duke of York had 10 000 men. He lost 20% of them on the way to the top of the hill and he lost 5% of the rest as he marched them back down the hill. Who had the most men left?

### **Hints to support students:**

How many soldiers did the Old Duke lose on the way up the hill?

How many soldiers did he lose on the way down?

How many soldiers did he lose altogether?

Can you work out how many soldiers the Young Duke had in the same way?

### **Solution:**

The Old Duke:

10% of 10 000 is 1000 so the Old Duke now only has 9000 soldiers

15% of 9000 is 1350 so he now only has 7650 soldiers

The Young Duke:

20% of 10 000 is 2000 so the Young Duke now only has 8000 soldiers

5% of 8000 is 400 so the Young Duke now only has 7600 soldiers

The Old Duke has 50 more soldiers than the Young Duke.



## The Grand Old Duke of York

**Topics covered:** Order of percentages

**Prerequisite knowledge:** Finding a percentage of an amount

**Problem:**

The Grand Old Duke of York had 10 000 men. He lost 10% of them on the way to the top of the hill and he lost 15% of the rest as he marched them back down the hill. What percentage of the 10 000 men were still there when they reached the bottom of the hill?

**Hints to support students:**

What information do you have?

How many soldiers did the Old Duke lose on the way up the hill?

How many soldiers did he lose on the way down?

How many soldiers did he lose altogether?

What percentage of the original 10 000 is this?

**Solution:**

10% of 10 000 is 1000 so the Old Duke has 9000 soldiers left

15% of 9000 is 1350 so he has 7650 soldiers left

$$\frac{7650}{10000} \times 100 = 76.5\% \text{ of the soldiers are left}$$



## Plants for sale...

**Topics covered:** Comparing percentages of amounts

**Prerequisite knowledge:** Finding percentages of amounts

**Problem:**

Mrs Gardener had 1000 bedding plants she was growing to sell. She then lost 20% of her plants in a frost. The following week the number of plants she had increased by 24% due to another set beginning to grow. Does Mrs Gardener now have more or less plants than her original 1000 and by how many?

**Hints to support students:**

What is the first thing you can find out?

How many plants were lost in the frost?

What fraction is 20%?

How many plants does Mrs Gardener have left?

How many new plants does Mrs Gardener get?

Is this more or less than she lost?

If she lost 20% and then gained 24% does she end up with 4% more?

**Solution:**

20% of 1000 plants is 200 plants so Mrs Gardener has 800 plants left.

24% of 800 plants is 192 plants so Mrs Gardener now has 992 plants.

She has 8 plants less than 1000.



## Shopping in sales...

**Topics covered:** Order of percentages

**Prerequisite knowledge:** Finding a percentage of an amount

**Problem:**

In a sale prices are reduced by 10%.

After the sale prices go up by 10%.

Are the prices the same as before? Explain your reasoning.

**Hints for students:**

Choose a price for the item and work out the percentages based on that price

Work out the two percentage changes separately

**Solution:**

10% of the whole is 10%, so taking it off from the whole price leaves 90% of the whole price

10% of the 90% price is 9%, so adding it on gives 99% of the whole price

The price is lower than before



## Unknowns...

**Topics covered:** using percentages to find unknown values, by using proportion or other methods. Informally: 'working back' from percentages

**Prerequisite knowledge:** Finding percentage of value, percentage increase and decrease, converting from vulgar fraction to percentages, direct proportion and ratio, forming and solving equations

### Problems:

- 1 I have some money in a box. 30% of the money is £36. How much money is in there in total?
- 2 Mike drank 60% of his glass of milk. Afterwards, 80 ml of milk remains in the glass. What volume of milk was originally in the glass?
- 3 When a barrel is 30% empty it contains 30 litres more than when it is 30% full. How many litres does the barrel hold when full?
- 4 Part of a bridge spans a river 35 metres wide. One third of the length of the bridge is on one side of the river and 20% of the length of the bridge is on the other. How long is the bridge?

### Hints for students:

Can you write the percentages in a different way?  
How could you put all the information you have together?  
What could you use to represent a value you don't know?  
Do you have enough information? How do you know?  
Could you draw a picture?

### Solutions:

- 1 10% is £12 so 100% is £120
- 2 40% is remaining which is 80ml; 10% is 20 ml so 100% is 200 ml
- 3 70% is 30 litres more than 30% so 40% is 30 litres; 10% is 7.5 litres so 100% is 75 litres.
- 4 Changing to fifteenths...  $\frac{1}{3}$  is  $\frac{5}{15}$  and 20% is  $\frac{3}{15}$  so span is  $\frac{7}{15}$  which is 35 m; so  $\frac{1}{15}$  is 5 m so whole bridge is 75 m.





## Football match...

**Topics covered:** Finding amount remaining after a percentage reduction

**Prerequisite knowledge:** Finding a percentage of an amount

### Problem:

- Part 1 A crowd of 10 000 is expected for a football match:
- 10% get lost on the way to the ground and are late.
  - A further 20% are held up at the entrance when the ticket machines fail.
  - What % of the expected crowd got into the ground on time?
- Part 2 Another week, again a 10 000 crowd of supporters is expected for a football match.
- 25% are delayed by a train breaking down on the main line and are late.
  - A further 4% are held up at the entrance when a gate gets stuck.
  - What % of the expected crowd got into the ground on time?
- Part 3 Compare your answers and comment.  
Can you explain this?

### Hints for students:

How many were late because they got lost?  
So how many arrived at the ground on time?  
How many got delayed by the ticket machines?  
If 10% are late, what percentage is on time?  
Can you use this to find a simpler way of finding how many people got to the ground on time?

### Solution:

Part 1 10% got lost:  $10\%$  of  $10\ 000 = 1\ 000$ , so  $9\ 000$  got to the ground in time.  
20% got delayed by the ticket machines:  $20\%$  of  $9\ 000 = 1\ 800$   
So  $7\ 200$  got into the ground on time; this is  $72\%$  of the expected crowd

Part 2 25% delayed by train:  $25\%$  of  $10\ 000 = 2\ 500$  so  $7\ 500$  arrived at ground on time  
4% delayed by gate:  $4\%$  of  $7\ 500 = 300$   
So  $7\ 200$  got into the ground on time; this is  $72\%$  of the expected crowd

Part 3 Same result. *Encourage students to look at using a multiplier to get the amount after a reduction.*

$$90\% \text{ of } 10\ 000 = 9\ 000$$

$$80\% \text{ of } 9\ 000 = 7\ 200$$

$$\text{Or } 80\% \text{ of } 90\% \text{ of } 10\ 000$$

$$75\% \text{ of } 10\ 000 = 7\ 500$$

$$96\% \text{ of } 7500 = 7\ 200$$

$$96\% \text{ of } 75\% \text{ of } 10\ 000$$

$$\text{Push the more able to}$$

$$\text{see } 90 \times 80 = 75 \times 96$$



## Bird food...

**Topics covered:** Finding amount remaining after a percentage reduction

**Prerequisite knowledge:** Finding a percentage of an amount

### Problem:

A farmer plants 50 000 seeds.

10% of the seeds are eaten by birds and a further 5% do not germinate (grow).

What percentage of the seeds grow?

### Hints for students:

If 10% are eaten, what percentage is not?

If 5% don't grow, what percentage does grow?

### Solution:

10% of 50 000 is 5 000 so 45 000 are left to grow

5% of 45 000 is 2 250 so 42 750 grow



## Tadpoles...

**Topics covered:** Finding amount remaining after a percentage reduction

**Prerequisite knowledge:** Finding a percentage of an amount

**Problem:**

A pond contains 1 000 tadpoles.

By the end of each day 20% of the tadpoles alive at the start of the day have been eaten by predators.

By the end of which day is the population halved?

**Hints for students:**

If 20% are eaten, what percentage is not eaten?

**Solution:**

End of day one: 80% not eaten = 800

End of day two: 80% not eaten = 640

End of day three: 80% not eaten = 512

[Hopefully it will be clear that after another day more than half will be dead so 4 days]

End of day four: 80% not eaten = 409.6

[While it is not ideal that this is whole number, it is less than 500]



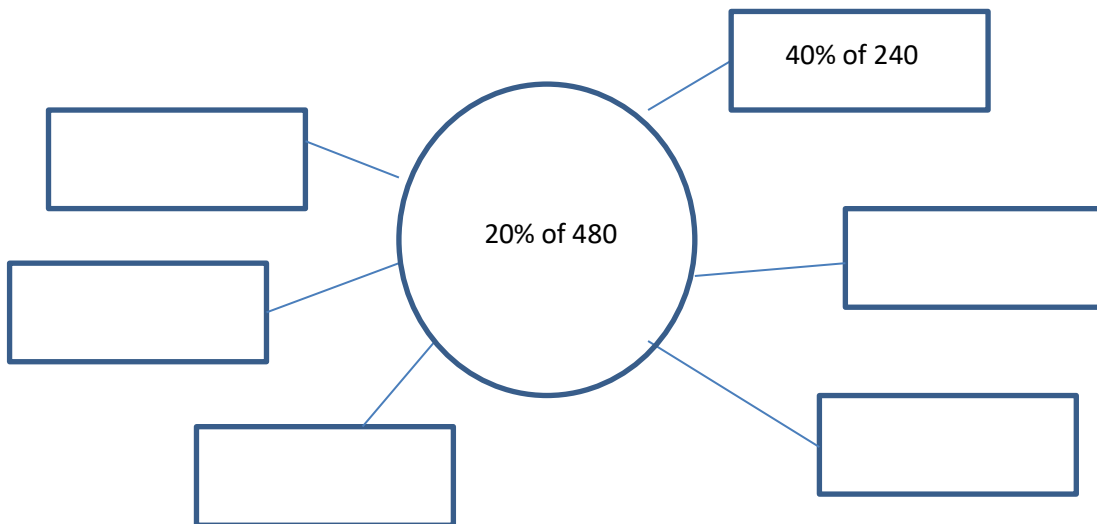
## Spider's Web...

**Topics covered:** Finding equivalent calculations

**Prerequisite knowledge:** Finding a percentage of an amount – this might be useful early in Year 7 as a review of prior knowledge and a way of introducing percentages greater than 100% or getting students to explore

**Problem:**

Can you fill in the other boxes?



**Hints for students:**

If you double the percentage what do you do to the amount?  
Look at the relationships between the percentage you are finding and the amount.

**Solution:**

There are many correct responses to this  
Consider altering the central question to differentiate



## Other places to look for problems:

[www.furthermaths.org.uk/enrichment](http://www.furthermaths.org.uk/enrichment)

[www.ukmt.org.uk/xxx](http://www.ukmt.org.uk/xxx)

[www.drfrostmaths.com](http://www.drfrostmaths.com)

[www.nrich.co.uk](http://www.nrich.co.uk)

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Betty Elkins and Lorna Piper



# Problems for photocopying



# Bus travel...

## Problem:

36% of the students at a certain school are mainly left-handed and all the other students are mainly right-handed.

25% of the left-handed students and 50% of the right-handed students at this school travel to school by bus.

What percentage of the whole school travel by bus?



## Giving orders...

### Problem:

The Grand Old Duke of York had 10 000 men. He lost 10% of them on the way to the top of the hill and he lost 15% of the rest as he marched them back down the hill.

The Grand Young Duke of York had 10 000 men. He lost 30% of them on the way to the top of the hill and he lost 5% of the rest as he marched them back down the hill.

Who had the most men left?





# The Grand Old Duke of York

## Problem:

The Grand Old Duke of York had 10 000 men.

He lost 10% of them on the way to the top of the hill and he lost 15% of the rest as he marched them back down the hill.

What percentage of the 10 000 men were still there when they reached the bottom of the hill?



## Plants for sale...

### Problem:

Mrs Gardener had 1000 bedding plants she was growing to sell.

She then lost 20% of her plants in a frost.

The following week the number of plants she had increased by 24% due to another set beginning to grow.

Does Mrs Gardener now have more or less plants than her original 1000 and by how many?



# Shopping in sales...

## Problem:

In a sale prices are reduced by 10%.

After the sale prices go up by 10%.

Are the prices the same as before?

Explain your reasoning.



# Unknowns...

## Problems:

- 1 I have some money in a box. 30% of the money is £36. How much money is in there in total?
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# Football match...

## Problem:

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Part 2 Another week, again a 10 000 crowd of supporters is expected for a football match.

- 25% are delayed by a train breaking down on the main line and are late.
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Part 3 Compare your answers and comment.  
Can you explain this?



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### Problem:

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# Tadpoles...

## Problem:

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By the end of each day 20% of the tadpoles alive at the start of the day have been eaten by predators.

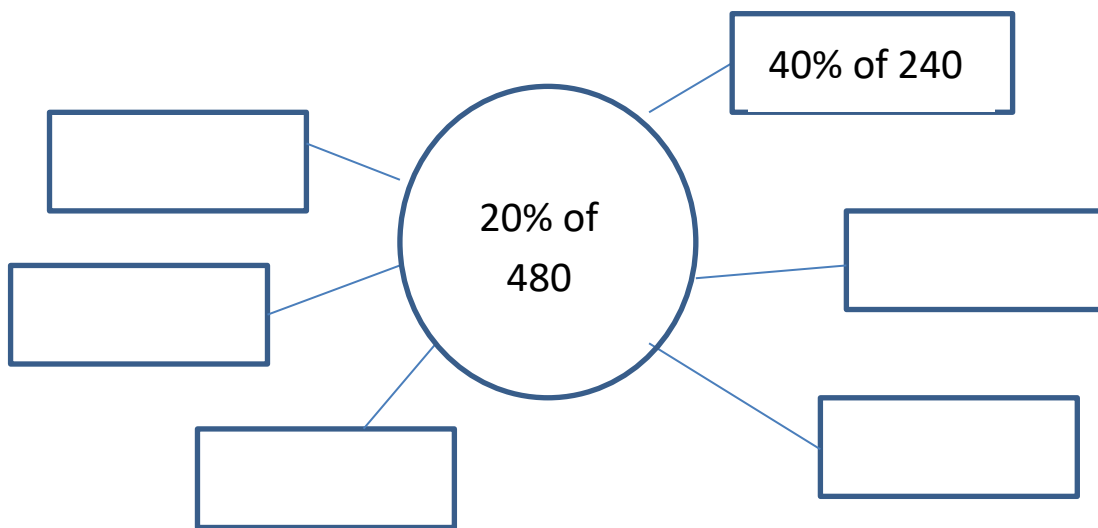
By the end of which day is the population halved?



# Spider's Web...

## Problem:

Can you fill in the other boxes?







**Hints to support students:**

Bus travel (easier)	If there are 1000 students in the school how many left-handed students are there?
Bus travel (easier)	How many left-handed students travel to school by bus?
Bus travel (easier)	How many right-handed students travel to school by bus?

Bus travel (harder)	Pick a number of students for there to be in the school. Now find the number of left-handed students in the school.
Bus travel (harder)	What do you think you might find out next?

Giving orders	How many soldiers did the Old Duke lose on the way up the hill?
Giving orders	How many soldiers did he lose on the way down?
Giving orders	How many soldiers did he lose altogether?
Giving orders	Can you work out how many soldiers the Young Duke had in the same way?

The Grand Old Duke of York	What information do you have?
The Grand Old Duke of York	How many soldiers did the Old Duke lose on the way up the hill?
The Grand Old Duke of York	How many soldiers did he lose on the way down?
The Grand Old Duke of York	How many soldiers did he lose altogether?
The Grand Old Duke of York	What percentage of the original 10 000 is this?



Plants for sale	What is the first thing you can find out?
Plants for sale	How many plants were lost in the frost?
Plants for sale	What fraction is 20%?
Plants for sale	How many plants does Mrs Gardener have left?
Plants for sale	How many new plants does Mrs Gardener get?
Plants for sale	Is this more or less than she lost?
Plants for sale	If she lost 20% and then gained 24% does she end up with 4% more?

Shopping in sales (easier)	What if the price was £100?
Shopping in sales (easier)	What is 10% off £100?
Shopping in sales (easier)	What is 10% of the sale price?
Shopping in sales (easier)	What is 10% of the sale price added onto the sale price?

Shopping in sales (harder)	Choose a price for the item and work out the percentages based on that price
Shopping in sales (harder)	Work out the two percentage changes separately



Unknowns	Can you write the percentages in a different way?
Unknowns	How could you put all the information you have together?
Unknowns	What could you use to represent a value you don't know?
Unknowns	Do you have enough information? How do you know?
Unknowns	Could you draw a picture?

Football Match	How many were late because they got lost?
Football Match	So how many arrived at the ground on time?
Football Match	How many got delayed by the ticket machines?
Football Match	If 10% are late, what percentage is on time?
Football Match	Can you use this to find a simpler way of finding how many people got to the ground on time?

Bird Food	If 10% are eaten, what percentage is not?
Bird Food	If 5% don't grow, what percentage does grow?

Tadpoles	If you double the percentage what do you do to the amount?
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Spider's Web	Look at the relationships between the percentage you are finding and the amount.
Spider's Web	If 20% are eaten, what percentage is not eaten?