Advanced Mathematics Support Programme

Factor Fun!

A quick recap

- List the first 8 prime numbers.
- What is the prime factorisation of 60?

Solutions

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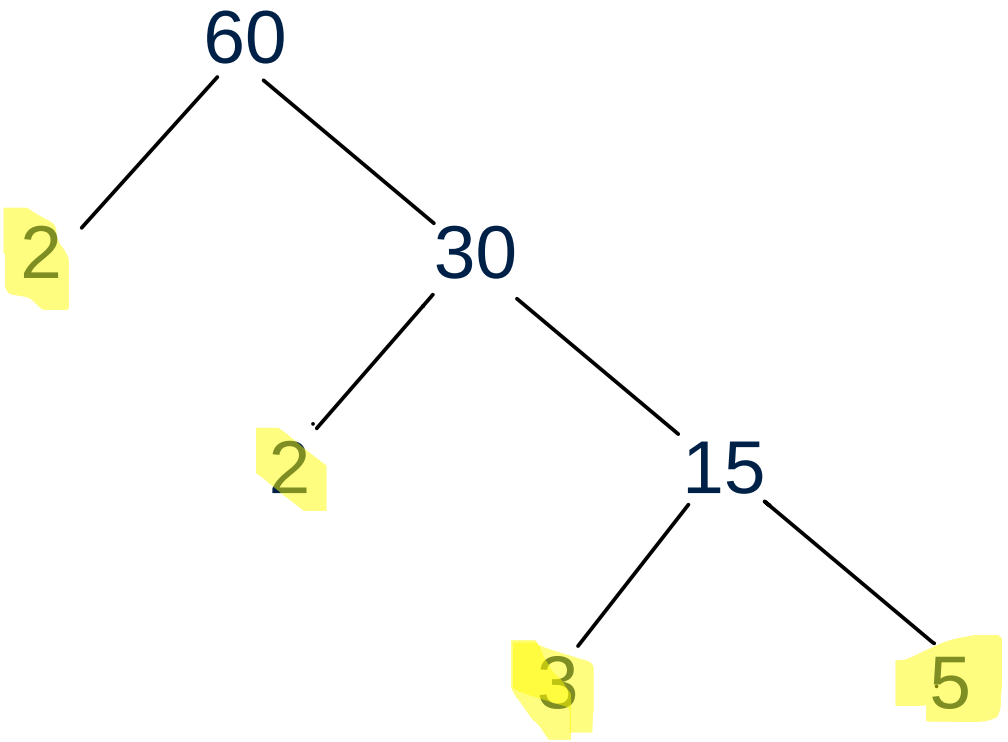
- The first 8 prime numbers are 2, 3, 5, 7, 11, 13, 17 & 19
- Prime factorisation is finding the prime numbers (factors) which multiply together to make the original number. This is the same as 'the product of prime factors'
- The prime factorisation for 60 is $2^2 \times 3 \times 5$

A factor tree!

You could draw a prime number factor tree for 60.



Three pupils Jess, James and Jem were asked to factorise the number **1656**

Each chose a different method, which one would you prefer? What are your reasons?

- **Jess chose**:

$2 \times 828 \rightarrow 2 \times 2 \times 414 \rightarrow 2^3 \times 207 \rightarrow 2^3 \times 3 \times 69 \rightarrow 2^3 \times 3^2 \times 23$

- **James chose**:

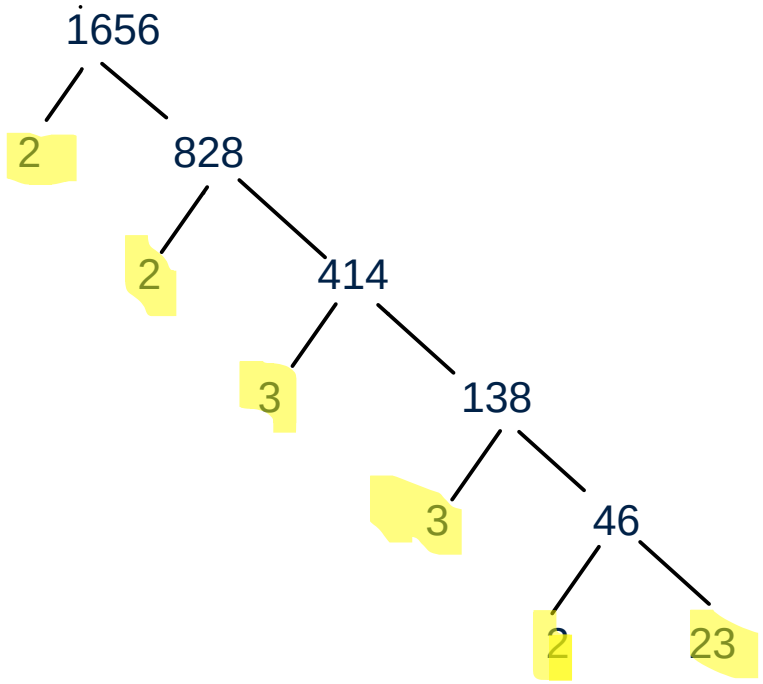
$3 \times 552 \rightarrow 3^2 \times 184 \rightarrow 2 \times 3^2 \times 92 \rightarrow 2^2 \times 3^2 \times 46 \rightarrow 2^3 \times 3^2 \times 23$

- **Jem started with**:

$6 \times 276 \rightarrow 2 \times 3 \times 276 \rightarrow 2^2 \times 3 \times 138$

$2^3 \times 3 \times 69 \rightarrow 2^3 \times 3^2 \times 23$

1656 – as a Prime Factor Tree



- Which method would you prefer? The prime factor tree or one of the methods chosen by James, Jess or Jem in the previous slide?

Time for an interesting fact!

If you find the prime factors of a composite number and list the exponents in the prime factorisation, you can find out how many factors the number has!

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Let's try it with 96

- Prime factorisation is $2^5 \times 3^1$
- Add one to each exponent $(5 + 1) \times (1 + 1) = 12$
- Number of factors of 96 = 12 which are…
- 1 & 96, 2 & 48, 3 & 32, 4 & 24, 6 & 16 and 12 & 8

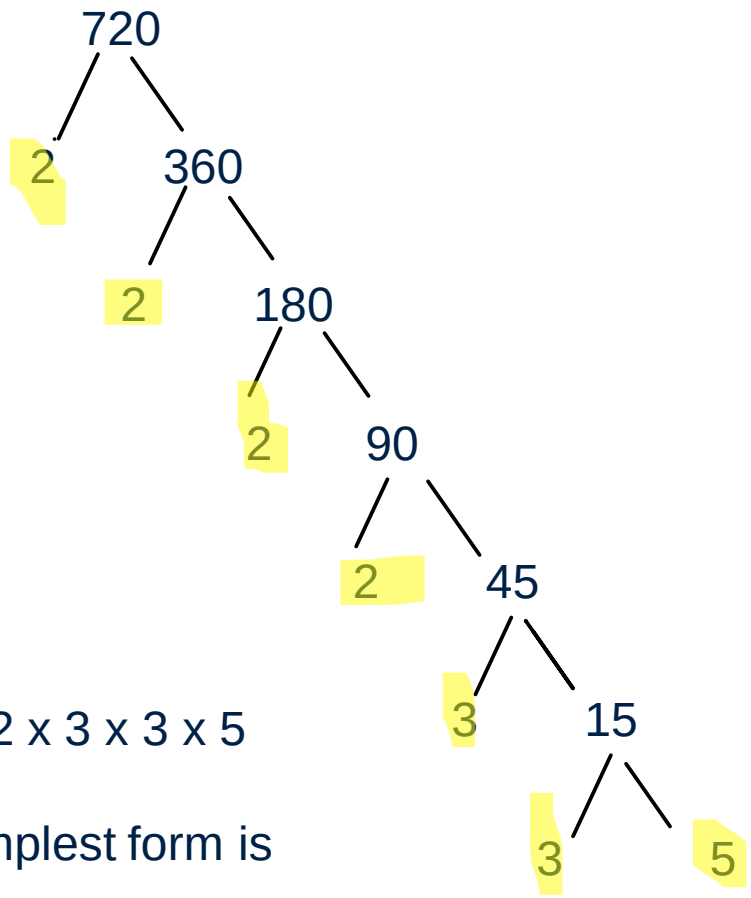
A simple enrichment task…

Try it with 720!

(Remember to add one to each exponent before you multiply them)

Check by listing the factors of 720 – were you correct?

Let's check our prime factorisation answer first!



Prime factors are

$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$

The answer in its simplest form is

$2^4 \times 3^2 \times 5$

Working out the number of factors…

The prime factorisation is $2^4 \times 3^2 \times 5$

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They are:

1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 30, 36, 40, 45, 48, 60, 72, 80, 90, 120, 144, 180, 240, 360, 720

(adapted from www.findthefactors.com)

Working out the number of factors…

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So the number of factors is $(4+1) \times (2+1) \times (1+1) = 30$

They are:

1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 30, 36, 40, 45, 48, 60, 72, 80, 90, 120, 144, 180, 240, 360, 720

You could try https://nrich.maths.org/countingfactors

Prime Factor fun

A certain number has exactly 8 factors including one and itself.

Two of its factors are 21 and 35

What is the number?

The solution is on the next slide

This investigation is taken from https://nrich.maths.org/4989

Prime Factor Fun (solution)

21 and 35 are factors
so 3, 7 and 5, 7 are prime factors

If 3, 5 and 7 are the only prime factors then the factors are:
1, 3, 5, 7,
15, 21, 35, and the number itself

The number = 3×5×7 = 105

Let's play with 210

The product of four different positive prime integers is 210

What is the sum of these four different prime integers?

Adapted from https://nrich.maths.org/7132

Let's play with 210 (solution)

Prime factorisation of 210 gives us 2 x 3 x 5 x 7

The sum of these prime numbers is 17

- Did you notice that 210 is the product of the first four prime numbers?
- And that the sum is also a prime number?

Is this true for other numbers?

Two for You!

1. The integer 113 is prime as is its reverse, 311.

   How many two-digit primes are there between 10 and 99 which have the same property?

2. Which numbers between 2 and 150 have the largest number of unique prime factors?

The solutions are on the next slides

Problem 1 is from nrich.maths.org/12584

Problem 1 (solution)

- 2, 3, 5, 7
- 11, 13, 17, 19
- 23, 29,
- 31, 37
- 41,43, 47
- 53, 59
- 61, 67
- 71, 73, 79
- 83, 89
- 97

These could be considered to be reversible

Problem 2 (solution)

You could list all the numbers from 2 to 150 and cross out the numbers with one prime number factor (all the prime numbers have one prime factor), then cross out all the numbers with two prime factors and so on. This could take some time!

It might be easier to do a bit of reasoning:

- 2 x 3 x 5 = 30, we cannot multiply this by 7 (the next smallest prime number) as this gives 210, which is too high.
- This means that the highest number of unique prime factors is 3.
- We can try the different combinations of three prime numbers
- There are eleven numbers with 3 unique prime factors: 30, 42, 66, 70, 78, 102, 105, 110, 114, 130 & 138

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