Advanced Mathematics Support Programme®

AMSP Professional development videos: Reasoning and proof

Aims of the session
■ To encourage teachers to reflect on their own practice and explore ways of helping students to understand reasoning and proof. ■ To provide teachers with the opportunity to use and create a bank of resources that can be trialled in future lessons.
Resources needed
■ A projector for showing the videos. ■ Internet access for the “Next Steps” task.

Introduction

Discuss the following questions:

- What difficulties do students face when trying to construct proofs?
- What can we do to help students overcome these difficulties?

There is now a greater emphasis on proof at A level, and many students struggle with both the rigour required and age-old “I’m not sure where to start!” problem. The videos in this section are designed to help teachers explore ways of getting their students to actively engage with proofs rather than just reading them from a book and trying to learn them.

Watch the **Proof by deduction** video, which shows a teacher helping his students to formulate algebraic proofs. After watching the video you might like to discuss the following questions:

- How does the structure of the task in the video help students to engage with the question?
- What methods could be used to assess students’ understanding of how to structure a proof?
- What would be the next steps in helping students to construct their own proofs?

Pair work

Task A is a card sort exercise similar to the one used in the video. The difference is that not all the cards are needed to construct the proof. In pairs, build the proof and identify the unnecessary and/or misleading cards.

Once you are satisfied with your solution, each take one of the proofs from Task B and create your own unnecessary or misleading steps. Write each step, including your own misleading steps, on the card template sheet and swap with a partner. Once you have tried each other’s cards give feedback:

- Was the level of difficulty appropriate for any A level student?

- Were there too many or two few red herrings?
- What questions might you ask a student to help them to reflect on the task?

These cards can then be typed up and used in future lessons with students. You could also try giving students incomplete sets of cards with blank cards to represent missing steps. Ask them to complete the proof and fill in the missing step(s) themselves.

Next steps

The other videos in this section provide further ideas for how to develop students' proof skills. The ***Cut the Knot*** website contains examples of proofs that link to different areas of the A level syllabus (and beyond), including some flawed proofs like the ones used in the Checking Proof video. Try to adapt some of these proofs into resources that could be used by students.

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Task A

Some of the following cards can be combined to construct a proof of the following fact:

"The mean of the squares of two real numbers is always greater than or equal to their product."

Assemble the necessary cards to make the proof and identify the unnecessary/misleading cards

$a^2 + b^2 \geq 2ab$	QED
$(a-b)^2 \geq 0$ since any square number is non-negative	$(a+b)^2 \geq 0$ since any square number is non-negative
Assume, without loss of generality, that $a > b$	Let a and b be real numbers
$a^2 - 2ab + b^2 \geq 0$	$a^2 + 2ab + b^2 \geq 0$
$\dfrac{a^2 + b^2}{2} \geq ab$	$a^2 + b^2 \geq 0$ since any square number is non-negative

Solution

Six cards were required. The proof is as follows:

Let *a* and *b* be real numbers

$(a-b)^2 \geq 0$ since any square number is non-negative

$$a^2 - 2ab + b^2 \geq 0$$

$$a^2 + b^2 \geq 2ab$$

$$\frac{a^2 + b^2}{2} \geq ab$$

QED

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Task B

Choose one of the following proofs (or a proof of your own) and create extra unnecessary or incorrect substitution steps. Copy all steps, including the red herrings, onto the card template and challenge a partner to select and assemble the correct proof.

Proof that the sum of three consecutive integers is a multiple of three.

- Let n be an integer
- Then n, n+1 and n+2 are consecutive integers
- $n + n+1 + n+2 = 3n + 3$
- $3n+3 = 3(n+1)$
- QED

Proof that dividing any three digit number with equal digits (e.g. 777) by its digit sum gives the answer 37.

- Let n be an integer such that $0<n<10$
- $100n + 10n + n$ is a three digit number with equal digits
- $100n + 10n + n = 111n$
- $n + n + n = 3n$
- $111n/3n = 111/3$
- $111/3 = 37$
- QED

Proof that the difference of two consecutive square numbers is always odd.

- Let n be an integer
- Then n^2 and $(n+1)^2$ are consecutive square numbers
- $(n+1)^2 – n^2 = n^2 + 2n + 1 – n^2$
- $n^2 + 2n + 1 – n^2 = 2n + 1$
- 2n is even
- 2n + 1 is odd
- QED

Proof that the sum of two consecutive odd numbers is the difference of two squares.

- Let n be an integer
- Then 2n+1 and 2n+3 are consecutive odd numbers
- $2n+1 + 2n+3 = 4n+4$
- $(n+2)^2 – n^2 = n^2 + 4n + 4 – n^2$
- $n^2 + 4n + 4 – n^2 = 4n + 4$
- QED

Blank card template
